Physical Conditions for Primitive Functional Hierarchies

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The origin of life problem is the context in which I began thinking about hierarchies. The origin of life is perhaps the most mysterious hierarchical interface of all, but at the same time I believe it may present one of the most instructive approaches to general hierarchical control problems. This is because the lower level pre-life processes are ordinary physics and presumably subject only to precise laws which do not include extra hierarchical rules or constraints. However, to be recognized as "alive" a collection of matter must exhibit some additional integrative function by exerting a collective control over the individual molecules. This integrative function is what characterizes hierarchical control.

Hierarchical systems raise two types of questions. Viewed from the upper level of the hierarchy the existing *constraints* are taken for granted and the significant question seems to be *How does it work?* The answer found from this perspective always amounts to the discovery that the parts obey the laws of the lower level. Thus we find that if we take apart a working machine, like a watch, there is no detail of motion which evades physical laws. People with this perspective often claim that molecular biologists have reduced life to physics since they have taken the cell apart like a watch and found that no detail evades physical laws 1 .

On the other hand, viewed from the lower level of the hierarchy it is the laws of motion themselves which are assumed to be inexorable, and the significant question seems to be *How* could the constraints arise? The answer usually given from this

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^{1.} All the authors on molecular biology I have read tacitly assume that the classical idea of a deterministic machine is a good physical analogy to living matter. e.g., D. E. Wooldridge, *The Machinery of Life*, (New York: McGraw Hill, 1966). F. Crick, *Of Molecules and Men*, (Seattle: University of Washington Press, 1966). No one, except Polanyi (1968), points out that machines are designed and built only by man, and are therefore a biological rather than a physical analogy.

perspective amounts to the conclusion that the constraints are not clearly derivable from the laws of the lower level (Polanyi 1968). To this extent reduction appears very nearly impossible, and this is why some of us find the existence of hierarchies such a mystery.

If anyone takes care to formulate both of these questions with more logical precision, then I think he will find that both answers are correct for their respective questions. I believe most of us here feel that the second question is far more significant than the first. Nevertheless, either of these answers alone has tended, for hundreds of years, to stimulate great disputes. Since these questions arise from disjoint perspectives, the arguments are often largely polemical. Of course we do not desire or expect to avoid arguments over this discussion. All I can say is that I am not at all satisfied with the claims of either side, that physics explains how life's constraints work or the claim that physics cannot explain how life's constraints arose.

THE CONCEPT OF HIERARCHY

To begin I shall limit my use of the idea of hierarchy to autonomous hierarchies; that is, to collections of elements which are responsible for producing their own rules, as with collections which are designed to have contrasted hierarchical behavior by an external authority. I want to talk only about what might be called natural hierarchies rather than supernatural hierarchies, such artificial as man-made or machines or "special creations" of any kind. Secondly, I shall assume that all my examples are a part of the physical world and that all the elements obey the normal laws of physics. This does not mean that I assume a reductionist attitude. The auestion of what reduction can mean will become clearer, I believe, only after we discover the necessary physical conditions for a hierarchical interface. Thirdly I shall limit my definition of hierarchical control to those rules or *constraints* which arise within a collection of elements, but which affect individual elements of the collection. This is the normal biological case where, for example, in society a set of laws is enacted by the collective action of the group but applied to individuals of the group; or in the development of the organism, the collective interactions of neighboring cells control the growth or genetic expression of an individual cell; or in the enzyme where collective interactions of many bonds control the reaction of an individual bond.

Finally we must recognize the essential characteristic of hierarchical organization that the collective constraints which affect the individual elements always appear to produce some *integrated function* of the collection. In other words, out of the innumerable collective interactions of subunits which constrain the motions of individual subunits, we recognize only those in which we see some coherent activity. In common language we would say that hierarchical constraints produce specific actions or are designed for some purpose.

Right here I shall stop my description rather abruptly, since in talking about "function" I have passed over the hierarchical interface which always causes so much argument. Let me return instead to the first three conditions for a hierarchy: (i) autonomy, that is, a closed physical system, (ii) elements in the system which obey laws of physics, and (iii) collections of elements which constrain individual elements. I want first to express these conditions in the language of mechanics so that we can see the implications of these conditions as simply as possible.

STRUCTURAL HIERARCHIES

Descriptions of nature using the language of physics usually satisfy our first condition of autonomy by assuming a closed system. In classical mechanics the elements or particles in this system are said to have a certain number of degrees of freedom, which is just the number of variables necessary to describe or predict what is going on. Our second condition is that the

particles of the system follow the laws of motion. Classically this means that given their initial positions and velocities at a given time, the trajectories of the particles can be predicted in the future or explained in the past with arbitrarily high precision. But if we are restricted to classical physics, there is no way in which the third condition can be satisfied because it requires a "collection" of particles which constrains individual particles. The implication here is that some particles join together in a more or less permanent collection; otherwise the "collection" would only be transient and would depend crucially on the initial conditions. It was one of the serious difficulties of classical physics that there was no inherent why collections dvnamical reason should ever form permanently. In quantum mechanics, however, the concept of particle is changed, and the fundamental idea of a continuous wave description of motion produces the "stationary state" or a local time-independent "collection" of atoms and molecules. Since these local collections are constantly being perturbed, they are not really permanent, but have lifetimes which increase with the energy of the interactions which hold them together, and decrease with the thermal energy which knocks them apart. Although there are several types of bonds between atoms and molecules we need to distinguish only two - the strong and the weak bonds. The structures held together by the strong chemical bonds will have lifetimes much longer than structures held by weak bonds.

So far our simple physical description is useful up to the level of polymers and crystals, but now we need to see how such collections can "constrain" *individual* monomers or atoms which make up these collections. Up to this point, our description of matter is "normal physics" at the level of atoms and molecules, but the concept of "constraint" begins to sound like we are introducing new rules. What is the physical meaning of a constraint? The concept of "equation of constraint" was in fact first necessary in classical physics because of the lack of any dynamical process to explain the permanent loss of degrees of freedom of collections of matter in solid bodies. Another type of constraint is the boundary condition which limits the values of certain degrees of freedom independently of the equations of motion - e.g., when a particle is confined by a box. Both solid bodies as well as walls of boxes could be considered as collections of particles which influence the motion of individual particles, and so they fulfill the second condition of our definition of hierarchy. But while we know that solids can form spontaneously from individual particles. constraints such as boxes are usually designed by experimenters with some "higher" purpose in mind, and in this case our first condition of autonomy would not be satisfied. However, it is the stationary-state solutions of the primarily quantum mechanical equations of motion which account for permanent constraints

From such apparently simple beginnings we can see the origin of what are often called *structural* hierarchies. The richness as well as the orderliness in all the natural patterns of collections of molecules and crystals could be described as a selective and more or less permanent loss of degrees of freedom among many elements. Many scientists and philosophers will assert on principle that such hierarchical structure is entirely reducible to quantum mechanics. As is often the case, those experts who actually study the details are seldom so easily convinced. For example, Cyril Smith (1968) has pointed out that new levels of structural hierarchies usually depend on the appearance of an imperfection in the old level. But what do we mean by an "imperfection"? Which imperfections lead to new levels of organization, and which lead to greater disorder?

FUNCTIONAL HIERARCHIES

In spite of the enormous complexity which we can find in structural hierarchies, there is still something missing. There is seldom any doubt that such structures are lifeless. What is missing is some recognizable "function". No matter how intricate a structure may be, permanence is not compatible with

the concept of function. Function is a process in time, and for living systems the appearance of time-dependent function is the essential characteristic of hierarchical organization. To achieve function by permanently removing degrees of freedom in a collection of elements would be impossible. Instead the collection must impose variable constraints on the motion of individual elements. In physical language these amount to time-dependent boundary conditions on selected degrees of freedom. Furthermore, the time dependence is not imposed by an outside agent, but is inseparable from the dynamics of the system. Such constraints are generally called non-holonomic (non-integrable), and have an effect which is like modifying the laws of motion themselves. For example, the enzyme is not just a permanent linear string of amino acids residues, nor a permanently folded three-dimensional molecule. An enzyme is a time-dependent boundary condition for the substrate, which through the collective interaction of many degrees of freedom controls a few degrees of freedom so as to speed up the formation of a strong bond. Nor is it the essential peculiarity of the enzyme that it is a very *complicated* dynamical system. Any system with that many degrees of freedom is dynamically complicated. What is exceptional about the enzyme, and what creates its hierarchical significance, is the simplicity of its collective function which results from this detailed complexity.

To put the problem of dynamical hierarchical control in a more general way, it is easy to understand how a simple change in a single variable can result in very complicated changes in a large system of particles. This is the normal physical situation. It is not easy to explain how complicated changes in a large system of particles can repeatedly result in a simple change in a single variable. It is this latter result which we interpret as the "integrated behavior" or the "function" of a hierarchical organization. Thus, we find *structural* hierarchies in all nature, both living and lifeless, but we see *functional* hierarchies as the essential characteristic of life, from the enzyme molecule to the brain and its creations. However our recognition of *function* as having to do with a simple result produced by a complicated dynamical process is not useful unless we can give some physical meaning to the idea of simplicity. The problem is that the concept of simplification is not usually associated with the physical world, but rather with the observer's symbolic representations of this world. The world is the way it is. Only an observer can simplify it. In fact it is the assumption that the elementary motions are complete and deterministic that makes the generation of hierarchical rules appear so difficult. The hierarchical rule is superimposed upon a lawful system which is already completely deterministic. How can this be done without contradiction?

As far as I can see, this has never been done in physics without introducing what amounts to a measuring device or an observer. Unfortunately, since measuring devices and observers are usually associated with the brain, this does not resolve the contradiction, but only substitutes a human language hierarchy which is a harder problem than the one we are asking. I want to think of the most elementary configurations of molecules in which we recognize some simple objective function. So again the question arises: How can a lawful system of atoms which is maximally deterministic superimpose an additional functional rule or constraint upon its detailed motions?

And again, the only answer must be that the concept of functional constraint implies an alternative way of representing the detailed motions. But in a closed physical system there is no observer to represent the system in a different way. Therefore we are left with the idea that if we can recognize a simple hierarchical function in an isolated dynamical system, then we should also be able to recognize an internal representation or record of the system's own dynamics. Autonomous hierarchical function implies some form of self-representation. In other words, we may partially resolve the appearance of hierarchical order on an already completely ordered set of elements by saying that hierarchical rules do not apply to the elementary motions themselves but to a record of these motions. Before we

look at some examples of simple molecular collections which may exhibit internal records, let us see under what conditions our own hierarchic representations of physical systems arise.

DESCRIPTIVE HIERARCHIES

The hierarchical levels of our languages contain some of the deepest mysteries of logic as well as epistemology, but I believe they also contain a clue to the physical problem of the hierarchical interface. We have already mentioned the crucial interface between the strictly causal language of dynamics and the probabilistic language of statistical mechanics which has produced much distinguished controversy. I shall try to avoid the intricacies of the general arguments by using a simple example as an illustration.

When we speak of the elementary laws of mechanics we mean the laws that describe as precisely as possible how each degree of freedom changes in time, given the initial conditions and boundary conditions. These equations of motion are universal and apply to all detailed motions which take place in the system. In one sense, therefore, all additional information about the system is either redundant or contradictory. But if we are trying to describe, say, 10²³ molecules in a box, it is obvious that measuring or following each degree of freedom is impossible. However, as outside observers we have learned to recognize and define collective properties of molecules, such as temperature and pressure, which allow simple and useful measurements on the gas in the box. It is significant that these properties were measured long before their "molecular basis" was known, just as many hierarchical biological functions were accurately described before a "molecular basis" was discovered. In physics it was the later discovery of the molecular dynamics began the controversial attempts to reduce which thermo-dynamical description to mechanical description by rigorous mathematical arguments. Perhaps these attempts can be characterized as very nearly successful – but not quite. This result is not trivial, since "not quite proved" in mathematics is like "not quite pregnant" in biology.

We may look at the problem as arising from the inability of the formal mathematics to predict what collective properties of complicated systems will produce simple, significant effects in the physical world of the observer. In other words, while there is no question that the detailed equations of dynamics can be used to calculate previously well-defined averages or collective properties, there is no way to predict from only the dynamical laws of the system which definitions of collective properties are significant in terms of what we actually can measure. Thus in one sense we can derive the pressure in terms of a suitable average of dynamical variables, if we are given a precise definition of pressure; but this definition of pressure is not determined by the equations of dynamics. The concept of pressure appears useful only when the dynamical system is embedded in a particular type of observational environment.

More generally we may say that a physical system which appears complete and deterministic with the most detailed representation, can symbolic appear incomplete and probabilistic only with a new representation which relinquishes some of the detail. The new representation must therefore come about through the *combination* or *classification* of the degrees of freedom at the most detailed level so as to result in fewer variables at the new level. Formal reductionism fails simply because the number of possible combinations or classifications is generally immensely larger than the number of degrees of What must always be added to define a new freedom. representation is the rule of combination or classification which tells us how to simplify the details. In statistical mechanics this rule is usually a hypothesis of randomness or ergodicity, but the ultimate justification for any such rule is that it results in a more useful description of the system in the observational environment in which the system is embedded.

What can it mean, then, for a collection of particles to form an *internal* simplification or *self*-representation? What is the

meaning of an "observational environment" for a system which is closed? Clearly in an autonomous hierarchy there must be an internal separation of some degrees of freedom from other degrees of freedom which become constrained to impose collective and time-dependent boundary conditions on individual degrees of freedom. While we know such integrated systems exist in cells, and can design machines which operate in this way, we are still baffled by the spontaneous origin of this type of constraint.

It is, in fact, a characteristic difficulty of hierarchical interfaces in biological organizations that their actual operation may appear quite clear while their origin is totally mysterious. The genetic code is a good example of a crucial hierarchical interface that is clear in its operation, but mysterious in its origin. One might wonder, in fact, if there is some inherent reason why a hierarchical organization obscures its own origins. Since it is one general function of hierarchies to simplify a complex situation, Simon² has suggested that if there are "... important systems in the world that are complex without being hierarchic, they may to a considerable extent escape our observation and understanding." Putting it the other way around, I would also suggest that "being hierarchic" requires that the system control its dynamics through an internal record, which has some aspects of "self-observation."

THE LOWEST HIERARCHY

But this is only evading the question. Let us see if we can clarify the problem of hierarchical origins by looking at collections of molecules of gradually increasing complexity, watching closely for any signs of internal *classification* or *recording* processes which are the essential conditions for a

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^{2.} In addition to emphasizing the essential correlation between state and process languages in any functional hierarchy, Simon (1962) characterizes hierarchical organizations as "nearly decomposable" by which he means that the state space is larger than the trajectory space. This is nearly equivalent to what I call a non-holonomic constraint.

simplification of the detailed dynamics. If we can imagine such collections, then we may go on to ask if this internal simplification is inherently self-perpetuating, or if there appear to be additional conditions which must be satisfied to establish a persistent hierarchical organization of molecules.

Perhaps the simplest interesting level of complexity is crystal growth. First, consider an ideal, ionic crystal growing in solution. One might try to apply our hierarchical conditions by saying that the crystal surface, with its alternating positive and "classifies" the incoming ions, negative sites. and bv permanently binding each ion to a site with the opposite charge forms a "record" of the classification interaction. Now while this may be grammatically correct, it is really only a redundant statement. There is no real distinction here between the physical interaction of the ion and the binding site and what we have called the "classification" and "record" of this interaction. They are all the same thing. Furthermore, each ion's interaction is local and direct and does not involve the dynamics of any large collection of ions or any delay. Therefore, although we may call this ideal crystal an example of hierarchical structure. I would not say that it exhibits hierarchical control over its dynamics.

Let us go on, then, to a more realistic level. Consider crystal growth which is produced by an imperfection, such as a screw dislocation. This is a *statistical* process which requires more than one atom or molecule to be in metastable positions. In time these atoms would shift to stable positions if there were no further growth. But this screw-dislocation structure increases the rate of growth by many orders of magnitude, all the time maintaining its special structure even though the original collection which first introduced the dislocation has been buried deep within the crystal. In this example, I believe a much stronger case can be made in favor of calling this a kind of hierarchical control. First, the constraint which controls the growth dynamics is not simply the direct interaction between local atoms, but involves the *collection* of atoms which makes

up the dislocation. Second, this collection is not the original dislocation, but a *record* of a dislocation which is propagated over time intervals which are very long compared to the rate of addition of the individual atoms. However it is difficult to distinguish a classification process in this example since all the atoms are identical.

As a third more complicated example, then, imagine a protoenzyme made up of only two types of monomers in a linear chain. Suppose this particular sequence of monomers folds up into a catalyst which speeds up the polymerization of only one type of monomer. For this specific catalytic reaction to occur we must express the fact that the folded polymer can distinguish one type of monomer from the other, and on the basis of this distinction alter the dynamics of each correct type of monomer so that it reacts much faster. Or in other words, we may say that this sequence of monomers *classifies* its elements and *records* this classification by forming a single, permanent bond between monomers. Now is there anything wrong with calling this process a form of hierarchical control?

In so far as the polymer sequences are no longer determined directly by the dynamical laws of the individual monomers (including their inherent reactivities), but by the constraints of a special polymer which speeds up the formation of a particular sequence, this might be called hierarchical dynamics. But now I think we have some problems of autonomy. First, this specific catalyst was invented by me, and although we know such specific catalysts do exist as enzymes, my invention simply evades the origin problem, as well as the physical problem of how such specific catalysts work. However, I have in mind a problem which is much more important. I think this example misses the essence of hierarchical *control*. We may indeed have in the catalyzed homopolymer a kind of simple record of a rather complex dynamical interaction, but the record has no further effect.

The trouble is that in the context of autonomous hierarchies, what constitutes a "record" must be indicated within the closed

system itself and not by what I, as an outside observer. recognize as a "record." Obviously to generate autonomous hierarchical control the record must be read out inside the system. The time-independent constraints formed by the permanent strong bonds must in turn constrain the remaining degrees of freedom in some significant way. This was the case in the previous example of screw-dislocation crystal growth where the dislocation structure was both a record of a past collective imperfection and a catalyst for the future binding of individual atoms. Cyril Smith (1968) sees this process as requiring a new description somewhere in between the detailed dynamics of atoms and the simple, stationary averages of thermodynamics. He sees all complex structure as both a record and a framework: "... the advancing interface leaves behind a pattern of structural perfection or imperfection which is both a record of historical events and a framework within which future ones must occur."

Returning to the copolymer system, we see that it may indeed fulfill the function of a record of past events, but the homopolymer record which was catalyzed does not act as a framework for future events. To provide autonomous hierarchical control, the catalyzed product of one copolymer must lead to the catalysis of other specific reactions. Furthermore, if the record is not to be lost, each catalyzed sequence must in turn catalyze another, and so on indefinitely. Now clearly such a sequential process can be divergent or convergent depending on the rules of specificity for the catalyses. Even if we assume that there is no error in these rules, a divergent record would never be recognized. One might say, in this case, that the system's self-representation is as complex as the system itself. But I think no underconstrained system would produce such a chain of catalysts. The starting record would simply disintegrate.

Going back now to the hierarchical control in the screw-dislocation crystal growth, we may look at this example as the other extreme. Here the classification and record

possibilities are trivially *over* constrained. Since there is only one distinguishable type of monomer, there can be no classification and hence no linear record. The "record" is not distinguishable from the three-dimensional structure which is also the functional catalytic site. The same problem of *over* constraint could, of course, occur in a copolymer system where, say, an alternating-sequence polymer acts as a tactic catalyst for the same alternating sequence. But this is the point of these examples. I want to show that even the simplest hierarchical organization requires a balance between the number of degrees of freedom of its elements, the number of fixed constraints which function as a record, and the number of flexible constraints which encode or transcribe the record.

Of course from this simplest conceivable level of molecular assembly which exhibits a potential *classification-record-control* process, we should not expect to find the nature of hierarchical interfaces at all levels. Even these simple examples present unanswered questions. But in following the necessary physical steps leading from the dynamics of individual units to the collective control of individual units, I believe we can gain some insight into the spontaneous generation of hierarchical organization.

First, we see that the individual particles or units follow more or less deterministic laws of motion. These units were atoms or molecules in my examples, but we may also think of the units as cells, multicellular individuals, or population units. The "motions" of these larger units are not as deterministic as the motion of atoms, but they have definite patterns of unit behavior. Second, there are forces between units which produce constraints on the individuals. These forces cause permanent aggregations of units which act as relatively fixed boundary conditions on the remaining individuals. By "relatively fixed" I mean that the rate of growth or change of these aggregations is slow compared to the detailed motions of individual units. These strong forces form what we called structural hierarchies, but they are essentially passive constraints.

The third stage is crucial and, as we might expect, the most mysterious. If the fixed constraints are not too numerous, that is, if the aggregations are not too rigid, then weak forces become important in the internal dynamics of the aggregations and through this *collective* dynamics the aggregations can form time-dependent boundary conditions for the other individual units. This type of flexible or non-holonomic constraint reduces the number of possible trajectories of individual units without reducing the number of degrees of freedom. This amounts to a classification of alternatives which leads us to now use the higher language of information or control. The specific catalyst or enzyme is the simplest example of such a dynamical constraint; but at any level of hierarchical control where there are ordinary molecules which also act as messages, or where simple physical objects are said to convey information, there must be the equivalent of such dynamical constraints which classify alternative motions by leaving a record of their collective dynamical interactions.

As we said earlier, it is in the simplicity or relevance of these records or messages that we recognize hierarchical control; but how this simplicity originates remains a mystery. In practice, when a dynamically complex system exhibits simple outputs or records of its internal motions we switch languages from the detailed dynamical description to a higher language, which relinquishes details and speaks only of the records themselves. We might think of our simplified language as an effect necessitated by a system that is too complicated to follow in detail, as in the case of our thermodynamic description of a gas. On the other hand, in systems which exhibit autonomous hierarchical organization, it is the internal collective simplifications which are the cause of the organization itself. In this sense, then, a new hierarchical level is created by a new hierarchical language. Simon (1962) has come to a similar observing a broad class of hierarchical conclusion from organizations. He calls the lower level language a detailed "state description" and the upper level language is simple "process description." But the fact remains that whether it is the

system-observer interface in physics, the structure-function interfaces in biology, or the matter-record interface in the most primitive molecular hierarchies, these levels are presently established only at the cost of creating separate languages for each level.

CONCLUSION

I have described the simplest examples I can imagine of what might be called incipient molecular hierarchies. I have used only a rough, semi-classical language, and have not even touched on the crucial question of how specific catalysis or classification processes could be described in the deeper quantum mechanical language.³ Nevertheless, I find the physical concreteness of these simple examples very helpful in sorting out which conditions are most essential for establishing a hierarchical interface.

What we find is that even the lowest interesting example of a hierarchical interface is beset with precisely those difficulties that we find in all hierarchical structures, namely, that each side of the interface requires a special language. The lower level language is necessary to give what we might call the legal details, but the upper level language is needed to classify what is significant. As Polanyi (1968) has so clearly pointed out, living organizations are not distinguished from inanimate matter because they follow laws of physics and chemistry, but because they follow the constraints of these internal, hierarchical languages.

It is therefore difficult for me to escape the conclusion that to understand even the simplest biological hierarchies, we will have to understand what we mean by a record or a language in terms of a lower level language, or ultimately in terms of elementary physical concepts. Physicists have worried about the inverse problem for many years. In fact a large part of what is

^{3.} These questions are discussed elsewhere (Pattee 1967, 1969).

called theoretical physics is a study of formal languages, searching for clear and consistent interpretations of experimental observations. Biologists have never paid this much attention to language, and even today most molecular biologists believe that the "facts speak for themselves." Hopefully, as these facts collect, biologists, too, will seek some general interpretations. All these facts tell us at present is that life is distinguished from inanimate matter by exceptional dynamical or controls which constraints have no clear physical explanation. We will not find such an explanation by inventing new words for our description of each level of hierarchical control. Instead, we will have to learn how collections of matter produce their own internal descriptions.

ACKNOWLEDGEMENT

This study is supported by the Office of Naval Research Contract Nonr 225(90).

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PROCEEDINGS of the SYMPOSIUM held NOVEMBER 18-19, 1968 at DOUGLAS ADVANCED RESEARCH LABORATORIES Huntington Beach California

Edited by



Hierarchical Structures

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American Elsevier Publishing Company, Inc. NEW YORK · 1969